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Hash Based Digital signatures- A tutorial Review

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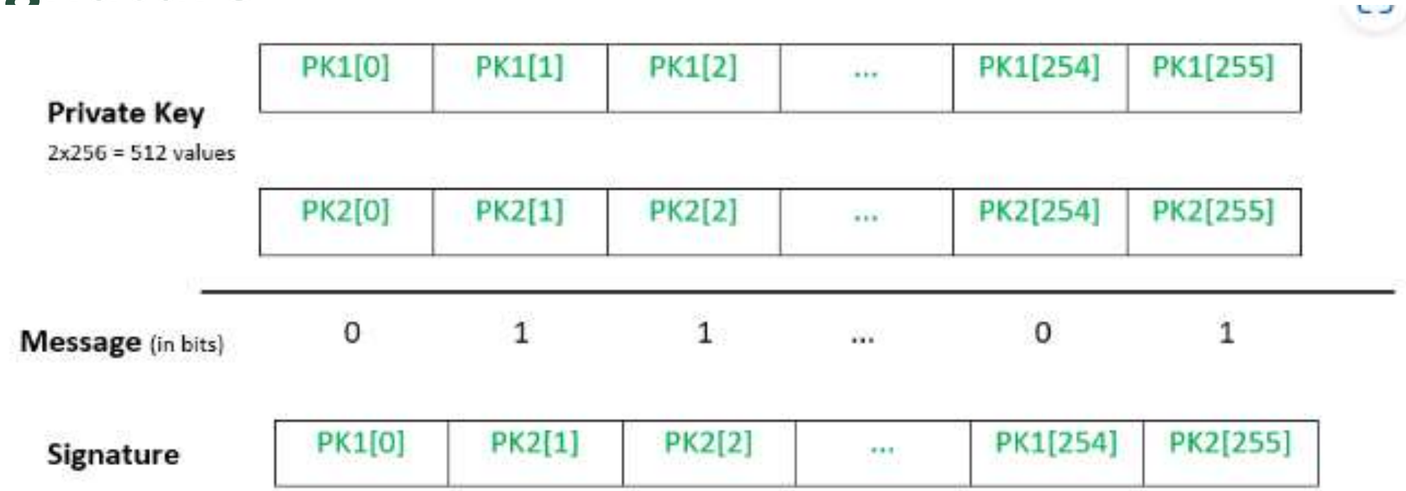
Lamport signature



- Two sets of 256 number of random 256 bit words S0 and S1. These form the **private key** of size: 512x256 bits.
- Hash both using SHA 256 to get two sets: H0 and H1 to get the **public key** of size: 512x256 bits.
- Hash your message to get a 256 bit hash (bits h0 to h255)
- If bit h(0) is 1 select from S1,0 ; else from S0,0 to get W0.
- If bit h(1) is 1 select fromS1,1 ; else from S0,1 to get W1 .
- Continue for all 256 bits of the message hash.
- Your hash is the obtained 256 number of 256 bit concatenated words: W0 ||W1||.....||W255.
- Hence Signature size is 256x256 bits = 8192 bytes.

Winternitz One Time Signature (WOTS)

•Signature size and Memory can be reduced by considering blocks of bits in the hash
•Block size is called **Winternitz parameter.** If w = 256, block size = log(w) = 8
• Private Key generation: Choose 32 random numbers each of 256 bits.
•Hash each of these **256 times** to get 32 number of 256 bit words. This forms the **Public Key.**
•Hash message to get 256 bit Hash. Consider as 32 bytes N(0)N(1)…N(31).
•Hash N(i) (256-N(i)) times. Repeat for each N(i) and concatenate all resulting 256 bit words. This forms the signature.
•Verification:
Verifier will hash the message to obtain N(0) N(1). ..N(31).
•Receiver hashes the received 256 bit words N(i) times . The result shall be same as the word in the public key corresponding to this block.
•Otherwise fails.

Winternitz One Time Signature

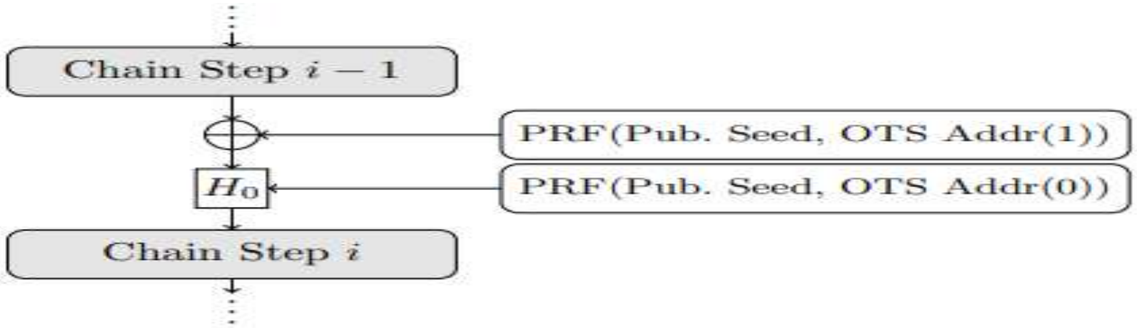
- Limitation: knowing the signature for a block m_i allows one to forge for any $m'_i > m_i$
- Solution: introduce a inverse check sum.
- Add all 32 number of (2^w-1-m_i) values
- Sum can be written a *l* additional blocks:

$$l_1 = \left\lceil \frac{n}{\log(w)} \right\rceil, l_2 = \left\lfloor \frac{\log(l_1(w-1)}{\log(w)} \right\rfloor + 1, l = l_1 + l_2$$

- Example: w=2, n = 256,l_1=128, l_2=5; *l* =133
- Thus 133 *Winternitz chains* will be required to do repeated hashings to get public key from private key and similarly for signatures.
- Size of w is trade-off between signature size and speed.

WOTS+

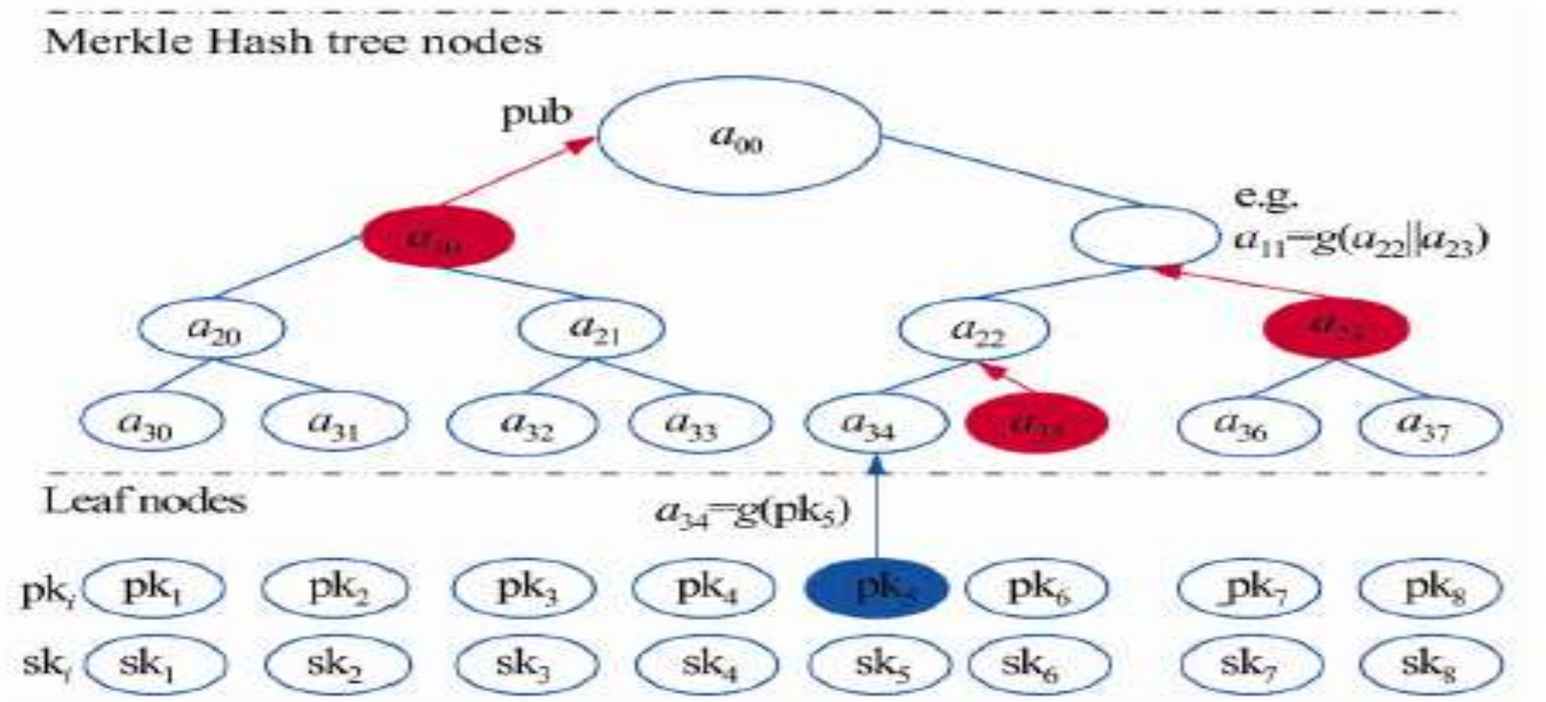
- Improvement over WOTS
- Uses a chain function instead of just hashing
- Needs Random bit masks ‘’r’’ to be generated for each step. *H* is a hash function,



- Limitation: Key size will be increased because of additional masks.
- Solution: generate them from a SEED all needed masks
- Used in XMSS (Extended Merkle signature system)
- One Pubic key, private key pair can be used only once for signing one message.

Extended Merkle Signature System

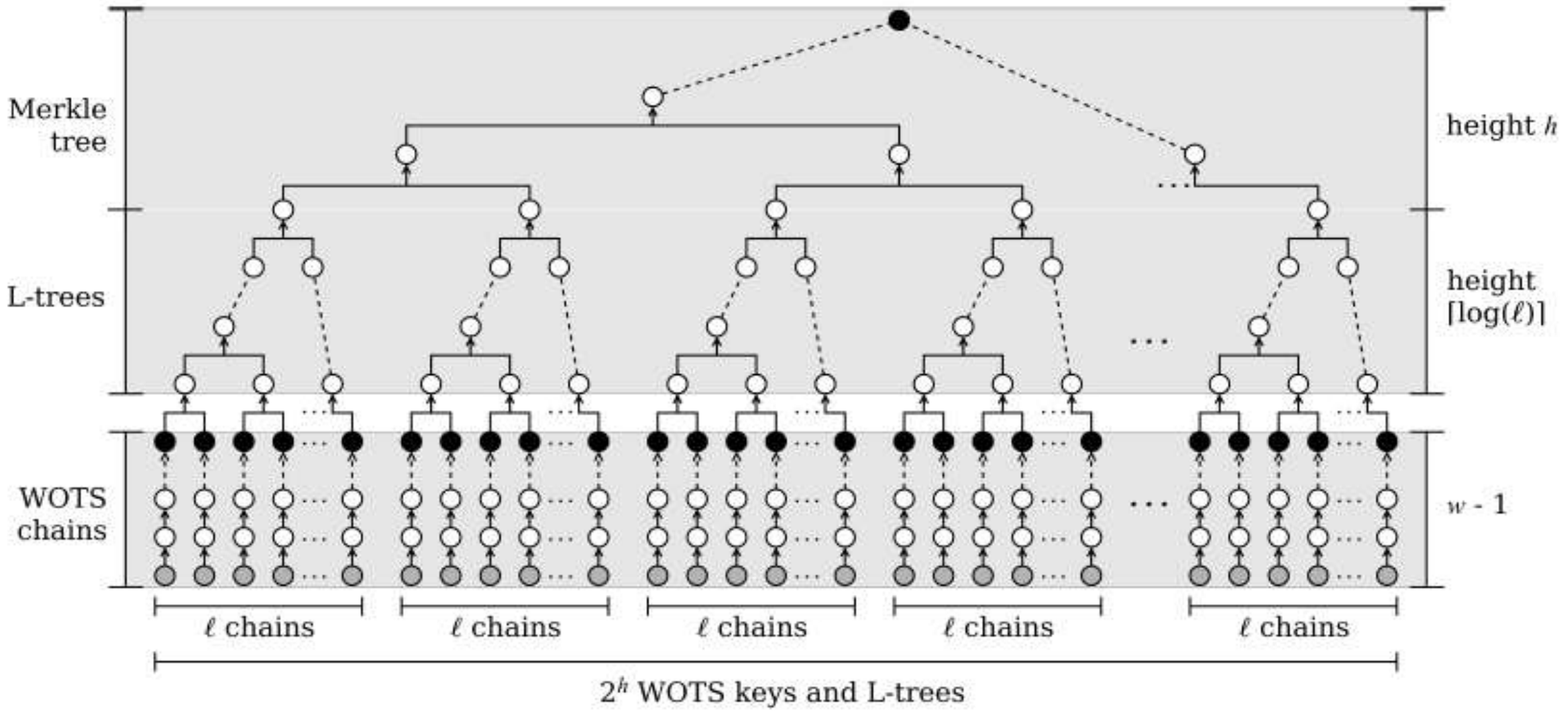
- *How to sign several messages? Use Merkle tree*



- *Verification path* $\{a_{35}, a_{23}, a_{10}\}$. .

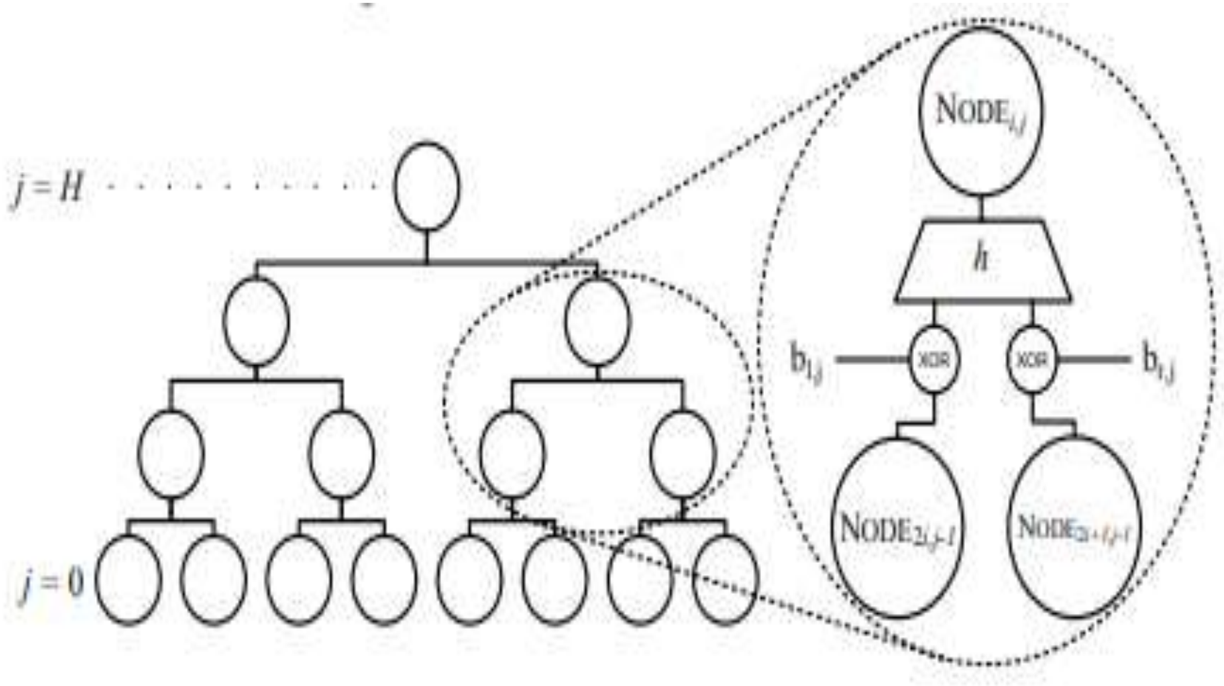
XMSS (RFC 8391)

- Uses Merkle tree and L-tree also. Can sign 2^h messages.
- Winternitz parameter *w* can be 4 or 16. Trade off: large *w* leads to smaller signature but more computation time.
- Black nodes are WOTS+ public keys
- Size of w is trade-off between signature size and speed.



L-tree and XMSS tree computation

•There are 2^h L-trees.
•L-tree is used to compress the WOTS+ public key ‘’len’’ number of *n* bytes each to n bytes. The levels needed are log (*len*).
•Generation of masks, additions (XOR), Hashing in each level of the tree is from a part of the public key seed 32 bytes using a PRF. Note PRF (pseudo random function) uses an additional input in addtion to actual input.

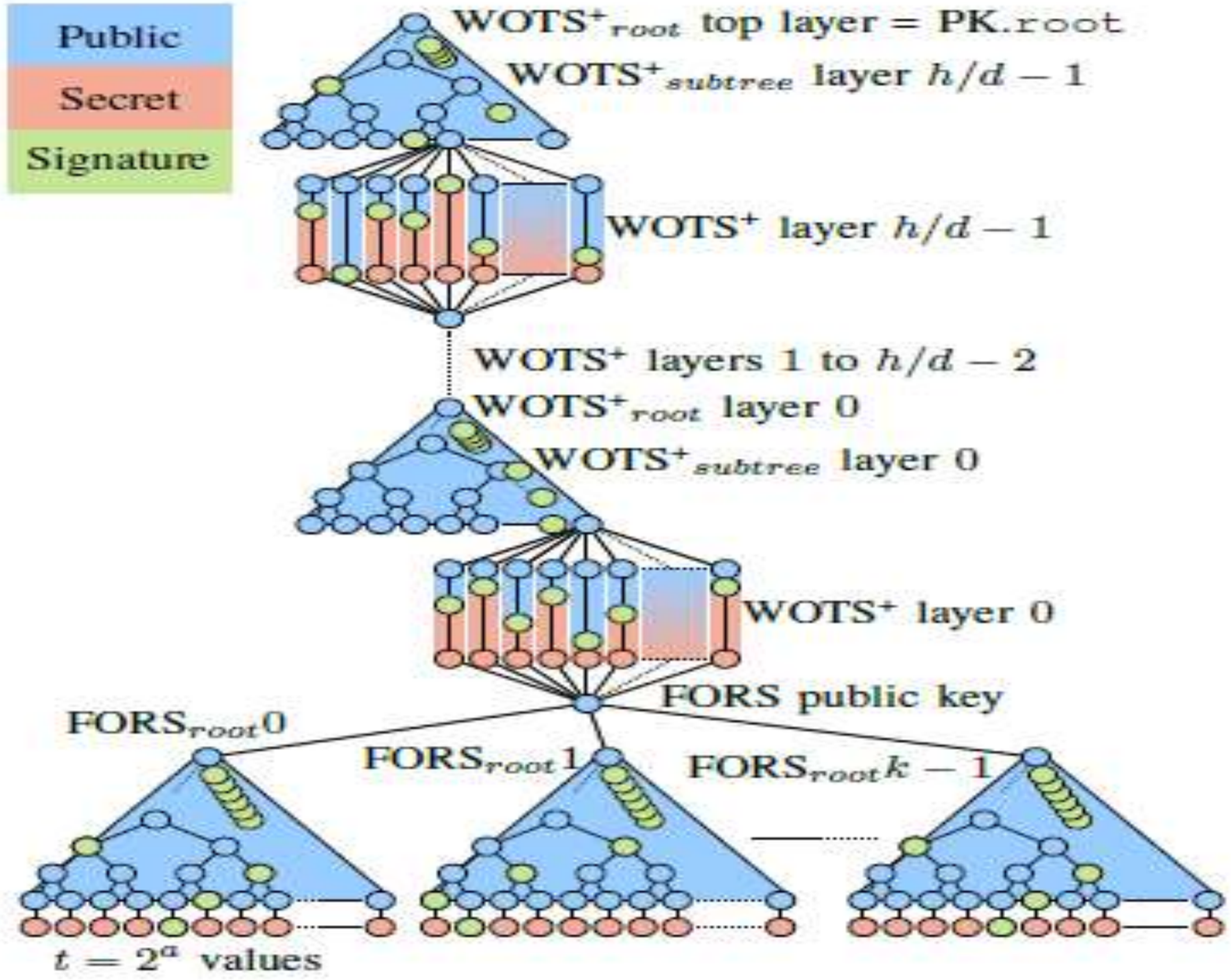


SPHINCS+

- **(Stateless Practical Hash-based Incredibly Nice Cryptographic Signatures)**
- Stateless: meaning no need to keep track of earlier usage/history
- Number of messages that can be signed = 2^{60} for each private key.
- Uses WOTS+, FORS (Forest of random sets) and variety of hash functions (SHA 256, Haraka, SHAKE, BLAKE and CHACHA stream cipher).
- Uses hierarchy of several trees in hypertree
- Several number of trees in FORS. (k trees)
- WOTS+ uses three options for w = 4, 16,256
- Security parameters available 128, 192 and 256 bit
- Resistance to multi-target attacks
- Uses tweakable hash functions, PRFs

SPHINCS+

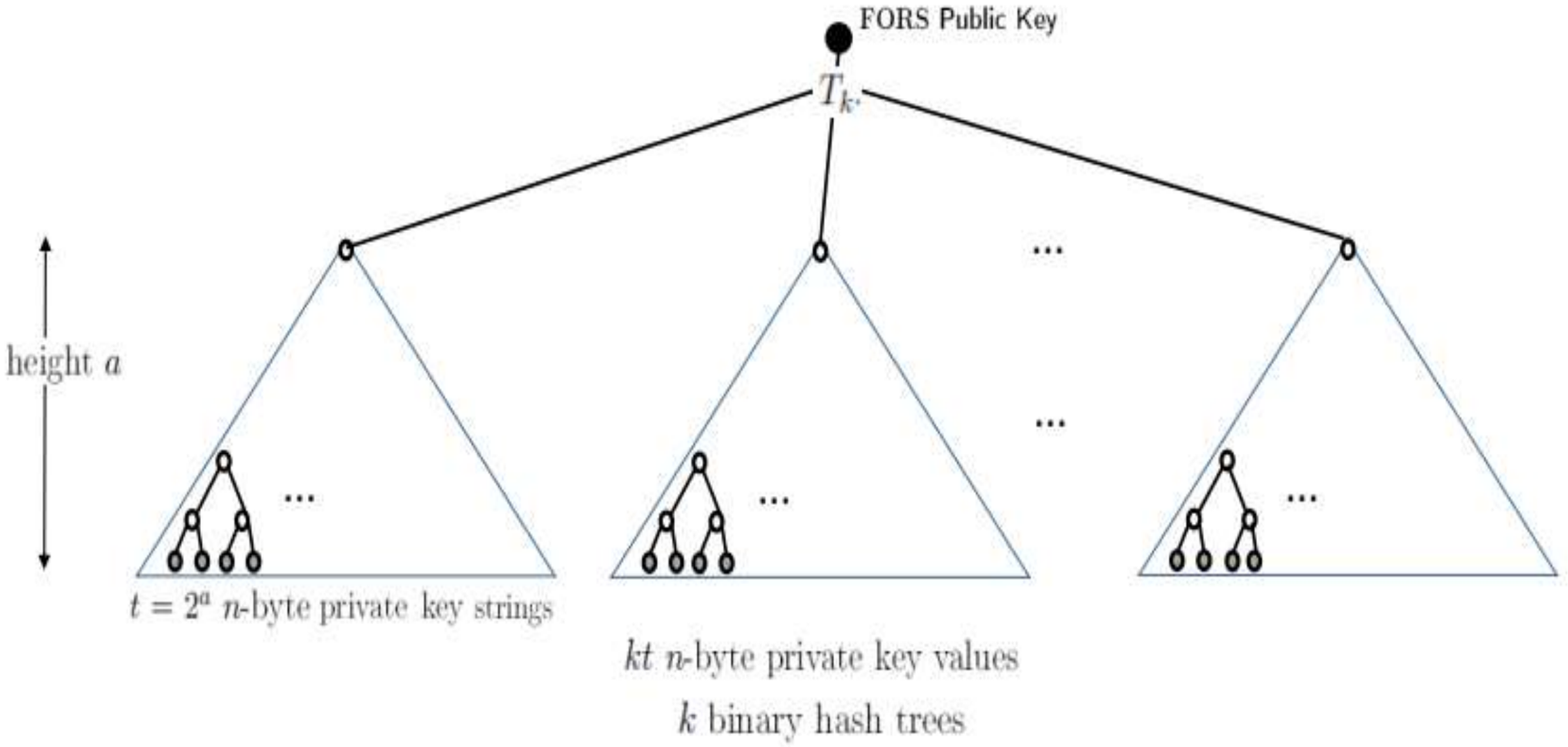
- Lowest layer is FORS
- Next WOTS+ layer signs the FORS root Public key.
- Intermediate trees sign the root of the tree below.
- There are enough leaves in the Merkle tree.
- It is statistically improbable to select same leaf for different signatures.
- Lowest layer signs FORS keys whereas each upper layer signs the public key of the immediate layer below.
- All trees have equal height.
- Total height = h; number of layers = d, height of XMSS tree is h' = h/d.



Parameters used in SPHINCS+

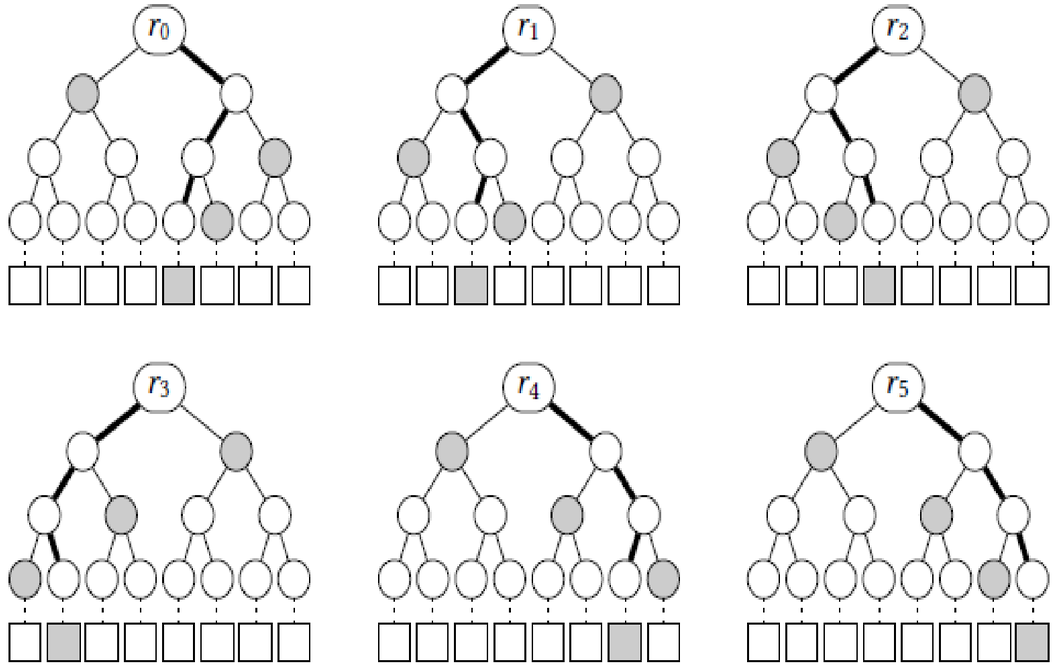
n	security parameter in bytes
h	height
d	number of layers of the hypertree
k	number of trees in FORS
t	number of leaves of a FORS tree

FORS (Forest of Random Sets) tree



- FORS is used to sign at random leaves

FORS illustration



- Randomizer R = PRF(SK.prf,optrand,M)
- MD‖idx = H_{msg}(R,PK.seed, pk.root. M)
- MD is fed to FORS where idx is the leaf index.
- Advantage is that the attacker cannot select an index freely.

- Message of k × a bits k = 6, a = 3 : **100 010 011 001 110 111.**

SPHINCS+ hypertree

- All leaf nodes of all intermediate trees are deterministically generated WOTS+ public keys that do not depend on any of the trees below it. This means that the complete **hypertree is purely virtual: it never needs to be computed in full.**
- **During key generation only , top-most sub- tree is computed .**
- Hypertree is not used to sign messages but is used to sign public keys of FORS instances.
- Similar to XMSS^{MT}
- It is a certification tree of XMSS instances
- SPHINCS+ generates a FORS instance and adds randomness to it for signing the original message. Then signs the FORS public key with an instance of the hypertree to get the SPHINCS+ signature.

Recommended parameter sets in SPHINCS+

	n	h	d	$\log(t)$	k	w	bitsec	sec level	sig bytes
SPHINCS$^+$-128s	16	63	7	12	14	16	133	**1**	7 856
SPHINCS$^+$-128f	16	66	22	6	33	16	128	**1**	17 088
SPHINCS$^+$-192s	24	63	7	14	17	16	193	**3**	16 224
SPHINCS$^+$-192f	24	66	22	8	33	16	194	**3**	35 664
SPHINCS$^+$-256s	32	64	8	14	22	16	255	**5**	29 792
SPHINCS$^+$-256f	32	68	17	9	35	16	255	**5**	49 856

- Example last entry: n=32, d=17,h=68,a=log (t)=9,k=35,
- we have signature = ($k \times n \times (a+1)$) +n + (h+($len \times d$)) $\times n$
- = (35x32x10)+32+ (68+(67x17))x32 = 11200+39648 = 49856 bytes

Comparison of Post-Quantum Round 3 Digital Signatures

Method	Public key size	Private key size	Signature size	Security level
Crystals Dilithium 2 (Lattice)	1,312	2,528	2,420	1 (128-bit)
Crystals Dilithium 3	1,952	4,000	3,293	3 (192-bit)
Crystals Dilithium 5	2,592	4,864	4,595	5 (256-bit)
Falcon 512 (Lattice)	897	1,281	690	1 (128-bit)
Falcon 1024	1,793	2,305	1,330	5 (256-bit)
Sphincs SHA256-128f	32	64	17,088	1 (128-bit)
Sphincs SHA256-192f	48	96	35,664	3 (192-bit)
Sphincs SHA256-256f	64	128	49,856	5 (256-bit)

CONCLUSION

- NIST statement for selecting SPHINCS+ in spite of large signature size and complexity
- **'If NIST's confidence in better performing signature algorithms is shaken by new analysis, SPHINCS+ could provide an immediately available algorithm for standardization at the end of the third round''.**
- Hash based signatures are robust.
- Even though these are very intensive in hash computations, efficient implementations of SHA2, SHAKE. SHA3 are available.
- More importantly, tomorrow if current hash functions are threatened, we can substitute with another improved/better hash function.

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THANK YOU

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